Exercise 31

Explain, in terms of linear approximations or differentials, why the approximation is reasonable.

$$\frac{1}{9.98} \approx 0.1002$$

Solution

Compute the derivative of $y = 1/x$.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$= -\frac{1}{x^2}$$

Consequently, the differential of $y = 1/x$ is

$$dy = -\frac{1}{x^2}\,dx.$$

In order to estimate $1/9.98$, set $x = 10$ and $dx = -0.02$.

$$dy = -\frac{1}{10^2}(-0.02) = \frac{1}{5000} = 0.0002$$

Note that dy here is the vertical distance from the function's actual value at $x = 10$ to the linear approximation's value at $x = 9.98$.

$$\frac{1}{9.98} \approx \frac{1}{10} + 0.0002 = 0.1002$$